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ABSTRACT

A multi-pass heuristic scheduling procedure developed for job shop scheduling problems with deterministic processing times is tested with processing times that are random variables. The heuristic procedure, which uses expected processing times, typically generates a delay schedule (i.e., a schedule in which some operations are delayed while the machine to process these operations is kept idle awaiting the arrival of another operation). Simulation is employed to compare the performance of the schedule generated by the heuristic procedure, a nondelay transformation of that schedule, and the nondelay schedules obtained with four single-pass dispatching rules. The criteria employed are fraction of jobs tardy, mean tardiness, variance of tardiness, and maximum tardiness. The delay schedule produced by the heuristic procedure was found to be markedly superior under certain conditions. Under other conditions, the relative performance of the scheduling rules appears highly problem dependent. Implications of these results are discussed in relationship to dynamic scheduling situations and man-machine interaction.

INTRODUCTION

In an earlier paper [6], the authors developed a multi-pass heuristic scheduling procedure for a common formulation of the job shop scheduling problem with job due dates. The formulation referred to is static and deterministic and may be briefly stated as follows: n jobs are available for processing in a shop that has m machines. Each job requires a given sequence of operations; each operation requires a specific machine. A machine can process only one job at a time. An operation time is given for each processing operation. Preemption of an operation in process is not permitted. Set up times and transportation times between operations are not considered. Each job has a due date representing a desired completion time for its processing. The problem is to sequence the processing of the operations on each machine to optimize some performance criterion related to job due dates.

The initial computational experience with the heuristic scheduling procedure (HSP) utilized a set of nineteen static and deterministic test problems for which schedules satisfying all job due dates were known to exist. HSP attained solutions to each of these test problems.

Statistical processing times for each processing operation[1] represent one important area that needs to be explored with regard to potential practical application in realistic scheduling situations. The authors' literature search and discussions have uncovered surprisingly little published work in this area [2,5,8].

1/ The distribution of actual processing time for a given operation used here should not be confused with the distribution of processing times for operations on a given machine used in queueing problems (e.g., exponential).

The research reported in this paper applies HSP and other due date oriented scheduling procedures to static problems with statistical processing times. The purpose is to explore the relative merits of a multi-pass (adjusting) scheduling procedure and single-pass dispatching procedures for a number of performance criteria and experimental conditions.

EXPERIMENTS

The experimental variables in the study were the priority rule, the processing time distribution, and the scheduling problem itself.

Priority rules. HSP, as described in [6], uses deterministic processing times and a multi-pass heuristic program to produce a schedule which, in general, is a delay schedule. Both HSP and a modification of HSP referred to as a nondelay transformation of the HSP schedule can be implemented with statistical processing times in the following manner: (a) To (centrally) use the HSP program with expected processing times to generate priorities for the operations on each machine; (b) To implement these priorities (locally) on the shop floor in conjunction with the actual processing times. The priority rule HSP implements the priorities in the order dictated by the centrally generated schedule even if available operations must be delayed. The priority rule HSP-NDT (the nondelay transformation of the centrally generated schedule) selects from among the operations actually available for processing on a given machine in priority order established by the centrally generated schedule.

The single-pass dispatching procedures considered for use in the experiments included two well-known due date oriented rules DDATE and SLACK, the shortest imminent processing time rule SPT, and four more refined procedures that have evolved during recent years. The latter consisted of Conway's [3] composite priority rule using a linear combination of SPT and SLACK per

operation, Oldziey's [9] dynamic composite rule in which the relative weights of several factors were varied with shop conditions to determine priorities, Carroll's [1] COVERT rule, and Malouin's [7] SPT truncated by SLACK per operation rule. All four of these rules have one or more arbitrary parameters. Experimentation varying these parameters has been with dynamic models and steady state conditions. For static problems of short time duration such as those used in this paper one would expect that optimal parameter values would be highly problem dependent. Because of this, it was felt that the behavior of these parametric rules with statistical processing times should be evaluated in a dynamic model. However, because of the importance of end conditions[2] in static problems, it was decided that a dispatching rule using both SPT and SLACK should be included as an example of a more refined rule that might actually be used in scheduling and appropriate for static problems. This rule, referred to as COMBINATION, uses SPT priority as long as that cannot result in any job in queue attaining negative SLACK as an immediate consequence. If the use of SPT priority can result in negative slack, priority is given to the job with the smallest value of [imminent processing time + SLACK]. The effect of this rule is to use SPT priorities early in the problem and to allow for more attention to job due dates

2/ i.e., the emptying of the shop with jobs approaching their due dates.

near the end of the problem by the inclusion of the SLACK term. The five priority rules used in the study are HSP, HSP-NDT, and the four single-pass dispatching procedures DDATE, SPT, SLACK (all as defined in [4]), and COMBINATION. Salient characteristics of the priority rules are summarized in Figure 1.

Priority Rule	Type of Scheduling Procedure	Type of Schedule	Used in Priority Determination	
			Processing Times	Due Dates
DDATE	single-pass	nondelay	no	yes
SPT	single-pass	nondelay	yes	no
SLACK	single-pass	nondelay	yes	yes
COMBINATION	single-pass	nondelay	yes	yes
HSP-NDT	multi-pass	nondelay	yes	yes
HSP	multi-pass	delay	yes	yes

Figure 1. Summary of Priority Rule Characteristics

Processing times. The description of the scheduling problem includes an expected processing time for each operation. Three distributions were used to obtain an actual processing time for each operation. The distributions were chosen to have different variances and to reflect the types of distribution forms that might occur in practice; (1) a uniform distribution over the range 80% to 120% of expected processing time, (2) a binomial distribution (with parameter p = .5) symmetric about the expected processing time, (3) a binomial distribution (with

parameter p = .1) skewed toward long processing times. The selected distributions result in processing time variances in the ratios 1:2.5:4.5 for the respective cases.

Problems. Three scheduling problems were selected for study on the basis of three problem characteristics which it was felt might affect the relative merits of the priority rules: problem size, tightness of due dates, and facility utilization. Figure 2 provides a brief description of the problems in terms of these characteristics.

Problem Number	Size	Using Expected Processing Times	
		Facilities Utilization	Tightness of Due Dates
1	12 jobs, 6 machines 57 operations	100%	HSP schedule satisfies due dates
2	6 jobs, 5 machines 30 operations	50%	
3	14 jobs, 7 machines 98 operations	65%	None of the schedules satisfies due dates

Figure 2. Scheduling Problems

Design and procedures. A three-factor complete factorial design with 6 x 4 x 3 = 72 factor combinations was employed. Eighteen of the factor combinations used the expected (deterministic) processing times. The 54 factor combinations with statistical processing times were each run for 50 independent replications. Validation of the simulator included chi-square

tests of the processing times generated for each distribution form and manual simulations to verify machine generated schedules for each priority rule. The schedule (Gantt Chart) and statistics were reported for each replication and summary statistics were recorded for the 50 replications. Although the focus of this paper will be the job tardiness statistics, the analogous data was obtained for job flow time and job lateness.

RESULTS

The fraction of jobs tardy, mean tardiness, variance of tardiness, and maximum tardiness for problems 1, 2, and 3 are given in Tables 1, 2, and 3 of the Appendix. For each of the above random variables, say x, the point estimate $\bar{x}$ of the population mean and the sample standard deviation s_x for the random sample of $n = 50$ observations are reported.

The point estimates of the population means for fraction of jobs tardy, mean tardiness, variance of tardiness, and maximum tardiness are graphed in Figures 3, 4, 5, and 6, respectively. The relative performance of the priority rules for each of the performance criteria, based upon these point estimates, are summarized in the following paragraphs.

Fraction of jobs tardy. For all three problems, the delay schedule based on the HSP priority rule is markedly superior when the variance in the processing times is small. As the variance in the processing times increases, the performance of

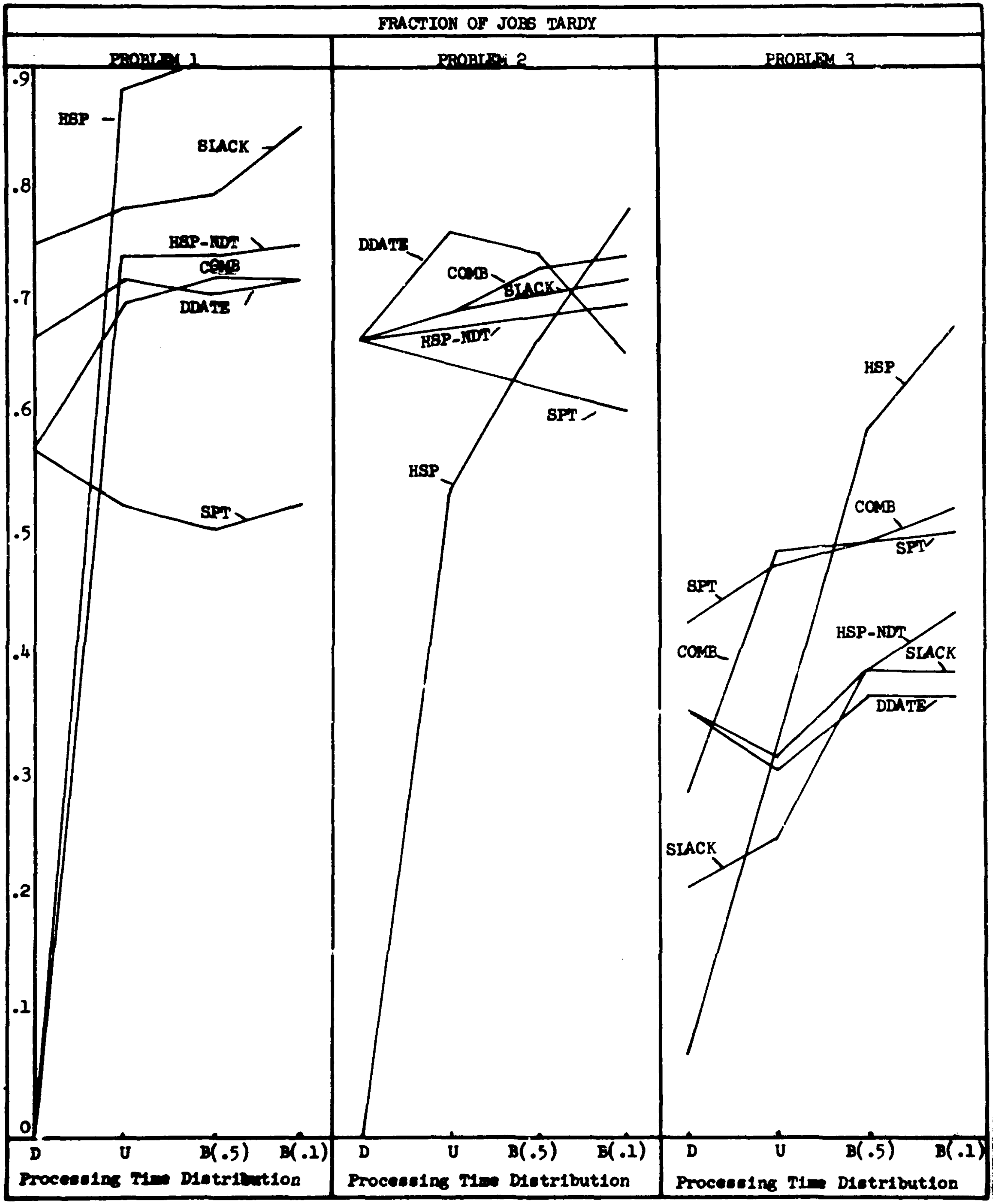

Figure 3. Point Estimates of Population Means for Fraction of Jobs Tardy

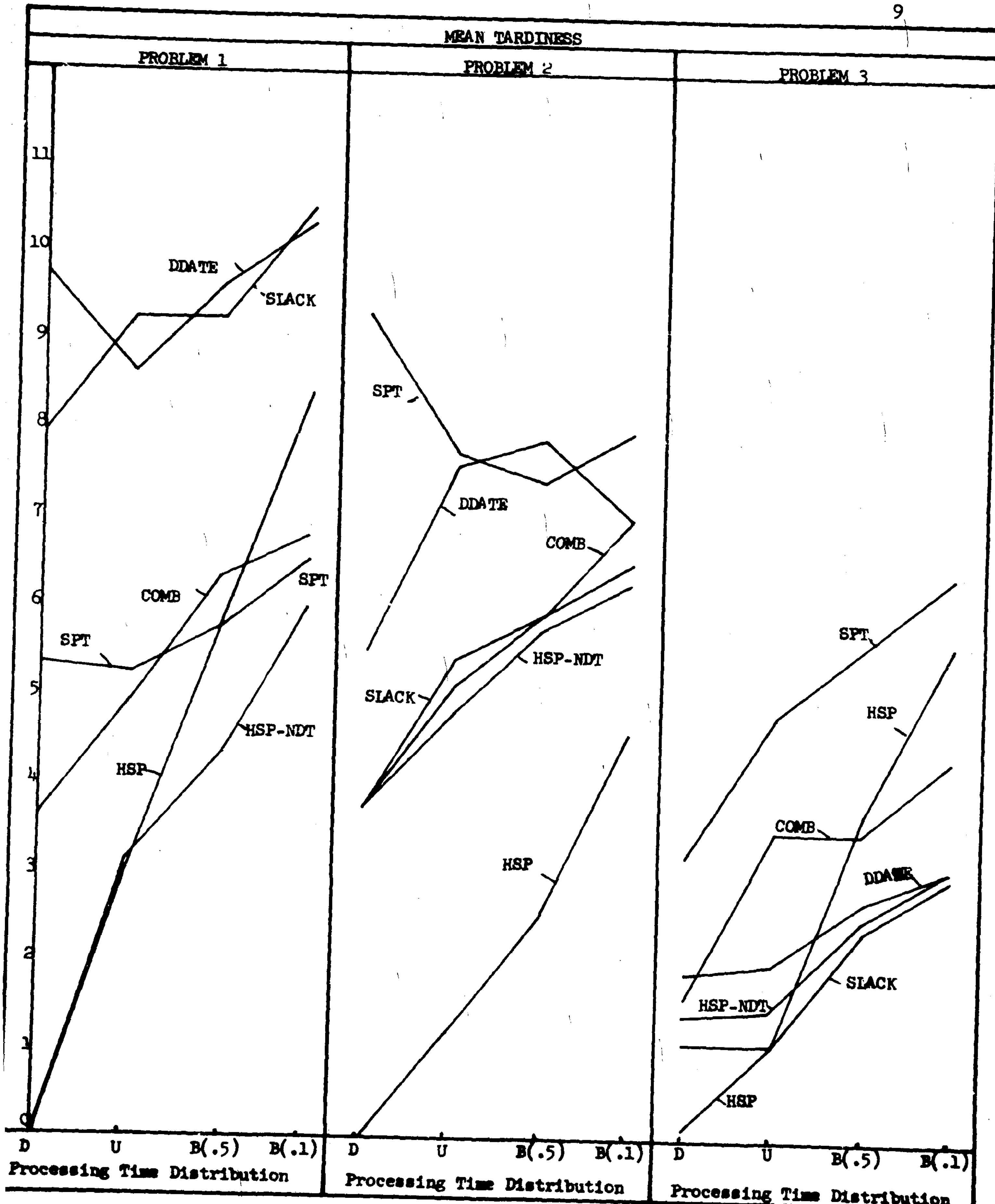

Figure 4. Point Estimates of Population Means for Mean Tardiness

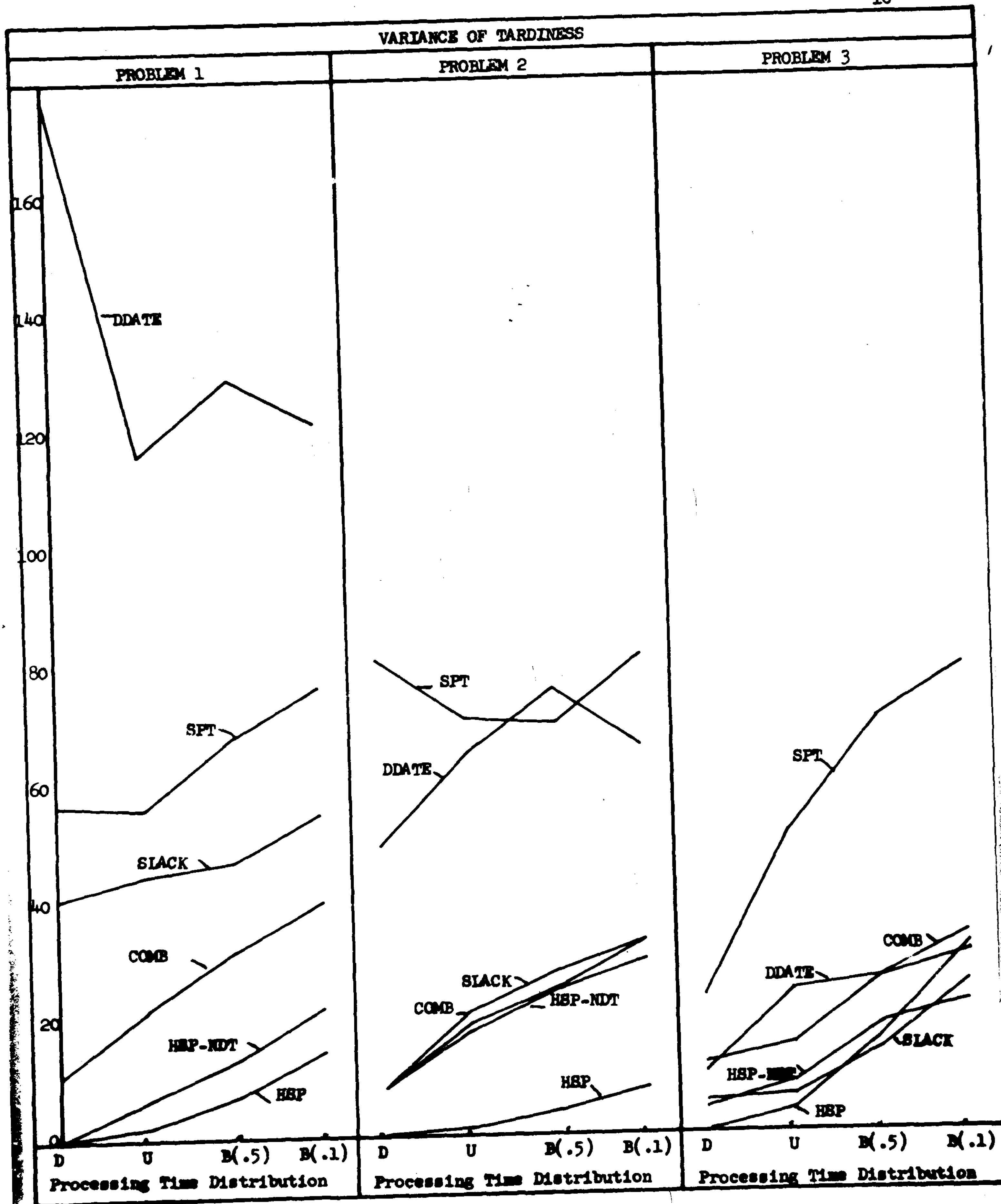

Figure 5. Point Estimates of Population Means for Variance of Tardiness

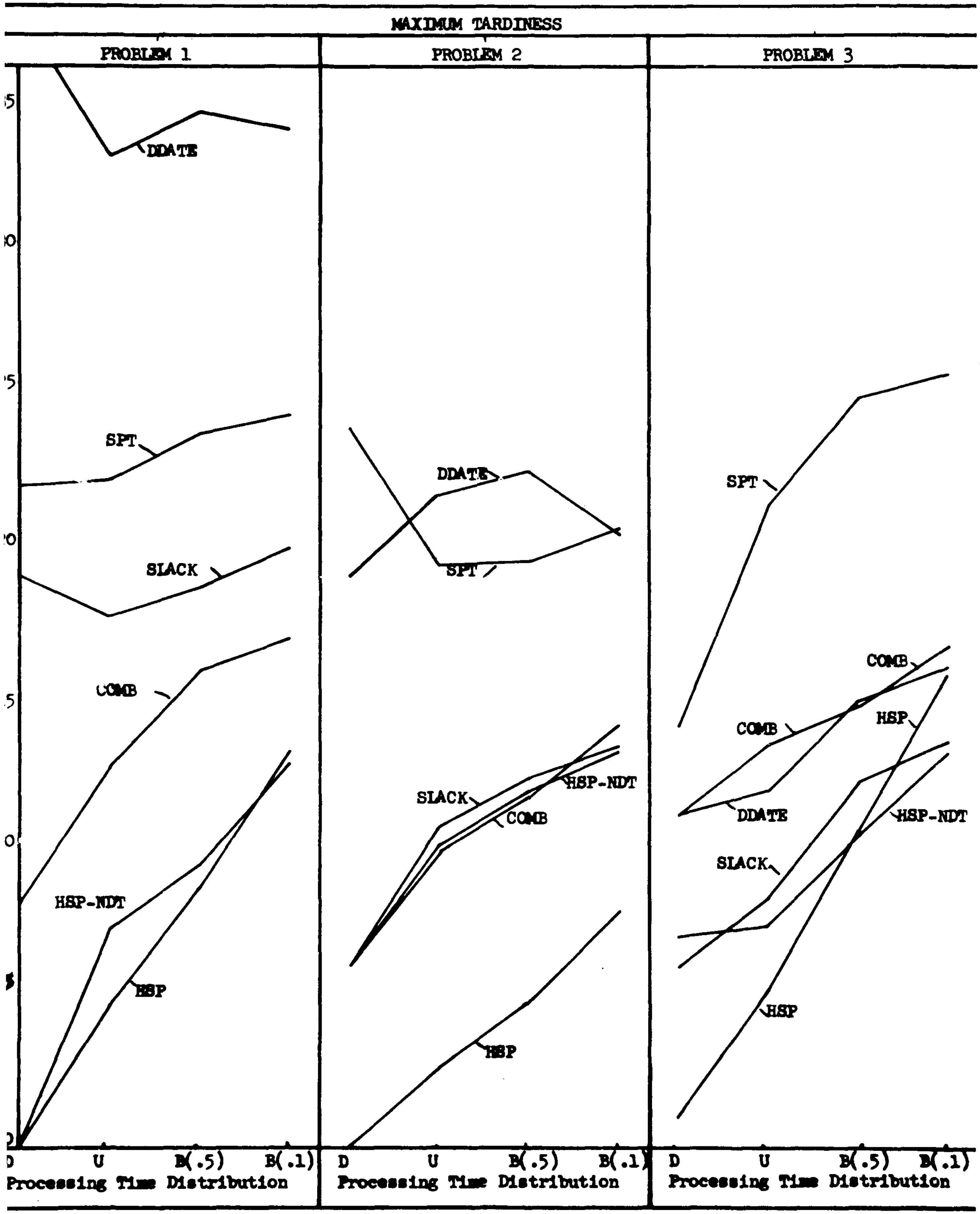

Figure 6. Point Estimates of Population Means for Maximum Tardiness

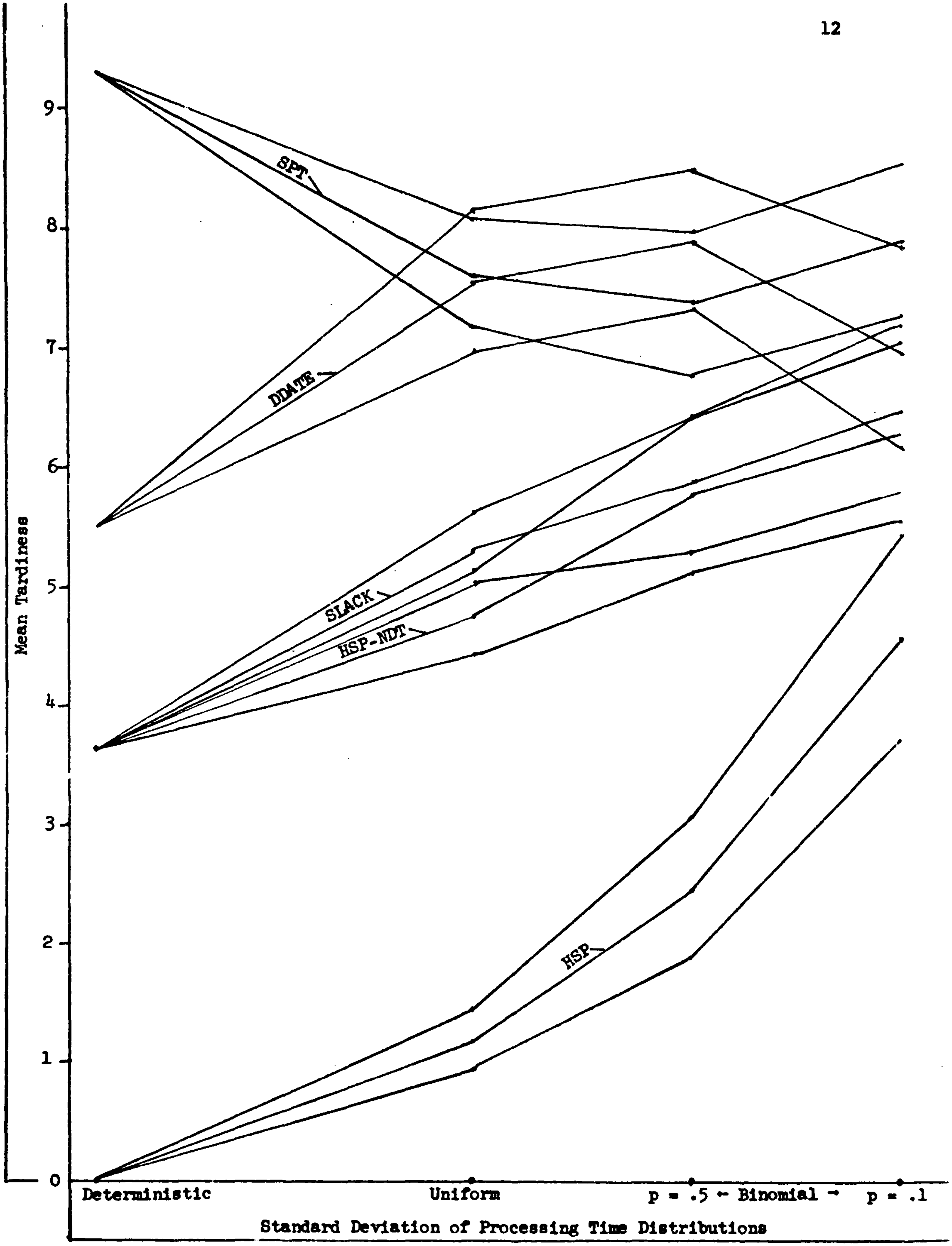

Figure 7. 95% Confidence Intervals for Mean Tardiness for Problem 2

HSP deteriorates quickly. With the high variance in the processing times, the relative performance of the five nondelay scheduling rules varies over the three different problems.

Mean tardiness. For all three problems, the delay schedule based on the HSP priority rule is markedly superior when the variance in the processing times is small. As the variance in the processing times increases, the performance of HSP deteriorates, but not as quickly as with respect to the fraction of jobs tardy criterion. Under conditions of high variance, the five nondelay scheduling rules appear to be competitive but HSP-NDT clearly gives the best average performance of the five -- taking into account the three different problems.

Variance of tardiness. Again, for all three problems, the delay schedule based on the HSP priority rule is markedly superior when the variance in the processing times is small. As the variance in the processing times increases, the performance of HSP deteriorates, but even more slowly than for the previously considered criteria. As with the mean tardiness criterion, HSP-NDT gives the best average performance of the five nondelay rules under high variance conditions.

Maximum tardiness. For all three problems, the performance of the rules is essentially identical to their performance with the variance of tardiness criterion.

The s_x values in the tables in the Appendix enable the computation of confidence intervals around the point estimates graphed in Figures 3 through 6. As an example, Figure 7 is a graph of the 95% confidence intervals for mean tardiness[3] for problem 2.

SUMMARY AND CONCLUSIONS

Summary of results. The principal results of the study based on the seventy-two factor combinations of six priority rules, four processing time distributions, and three problems may be summarized as follows:

(i) Perhaps the most important result of these experiments is the fact that the relative performance of the single-pass dispatching rules and HSP-NDT with deterministic processing times is a good indicator of their relative performance over the entire range of processing time variance introduced in the experiments.

(ii) The delay schedule based on centrally generated priorities using the multi-pass heuristic scheduling procedure (HSP) performed well, particularly for low variance in the processing times. The performance

3/ The confidence intervals for the COMBINATION priority rule would cluster with those of SLACK and HSP-NDT. They are omitted from the graph for clarity.

of the HSP delay schedule falls off with increasing variance in the processing times. The rate of deterioration and the relative performance with respect to the nondelay scheduling rules appears to depend upon the criterion being considered and upon the problem. In particular, the relative performance of the HSP delay schedule was best over a wider range of processing time variance for the higher moments (variance and maximum) of tardiness and for problem 2. Taken together, these results indicate that an heuristic procedure which produces delay schedules can make most effective use of enforced or planned idleness of facilities when actual processing times are fairly predictable and that, depending upon the problem and the criterion, relatively high performance may result even when actual processing times are highly variable. Thus, the use of HSP (or other delay schedule producing rule) merits serious consideration in practice.

(iii) Figures 3 through 6 illustrate that, when the variance in the processing times is large, the relative performance of the nondelay priority rules depends on the problem and the criterion under consideration. The nondelay transformation HSP-NDT was a consistently good performing rule under the conditions for which the HSP delay schedule was not. However, there are

indications that each of the single-pass dispatching procedures may perform better than HSP-NDT under some conditions.

(iv) The DDATE scheduling rule is the only rule tested that does not use information on processing times to compute priorities. Thus, it might be anticipated that the relative performance of DDATE would improve as the variance of the processing times increases. Figures 3 through 6 provide evidence to support this conjecture. Out of 26 crossings of the DDATE curves by those of other rules, 20 crossings are in an upward direction and 6 crossings are downward. The probability of 20 or more upward crossings for a binomial process with $n = 26$ and $p = .5$ is only .0046.

Implication and possible areas for additional research. The implication of the above results is that a centralized multi-pass scheduling procedure such as HSP merits serious consideration in job shop scheduling; either as a centralized machine procedure or in man-machine interaction. Under some conditions, the delay schedules produced by the procedure may be notably superior to nondelay schedules generated by single-pass procedures such as DDATE, SPT, SLACK, and COMBINATION. Beyond that, the easily implemented, nondelay transformation of the multi-pass procedure has been demonstrated to yield

relatively high performance over a wider range of conditions. Because there were also combinations of and criterion and experimental conditions for which one of the single-pass dispatching procedures was most effective, what is suggested is the use of an iterative procedure employing one or more single-pass dispatching rules to obtain an initial feasible schedule and then using the HSP program to attempt to improve upon the initial schedule. In view of result (i), the use of expected processing times in the iterative procedure should result in a good choice of schedule; providing that the variance of the actual processing times is not unusually large.

With respect to future research, the experiments with static problems and statistical processing times could, of course, be extended in scope beyond those reported here. More exciting, however, are the research prospects for dynamic problems with or without statistical processing times -- many of which can probably best be explored in an interactive mode. The authors are currently in the process of converting HSP to an interactive program for this purpose. The results of this study provide information which should be useful in planning later research.

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APPENDIX

Random Variable x >		Fraction of Jobs Tardy		Mean Tardiness		Variance of Tardiness		Maximum Tardiness	
Processing Time Distribution	Priority Rule	$\bar{x}$	s_x	$\bar{x}$	s_x	$\bar{x}$	s_x	$\bar{x}$	s_x
Deterministic	DDATE	.67	---	9.8	---	178	---	38	---
	SPT	.58	---	5.3	---	57	---	22	---
	SLACK	.75	---	7.9	---	41	---	19	---
	COMBINATION	.58	---	3.6	---	11	---	8	---
	HSP-NDT	0*	---	0*	---	0*	---	0*	---
	HSP	0*	---	0*	---	0*	---	0*	---
Uniform	DDATE	.72	.12	8.6	1.6	117	50.2	33.0	7.3
	SPT	.53*	.08	5.2	1.0	56	19.5	22.2	4.4
	SLACK	.78	.09	9.2	1.9	45	16.2	17.7	3.2
	COMBINATION	.70	.16	4.9	1.8	22	11.9	12.6	3.8
	HSP-NDT	.74	.18	3.1	1.2	7	4.6	7.2	2.2
	HSP	.88	.15	3.0*	1.2	2*	1.7	4.7*	1.5
p = .5 Binomial	DDATE	.71	.12	9.7	2.7	129	58.5	34.5	8.7
	SPT	.51*	.10	5.7	1.5	68	28.0	23.8	5.9
	SLACK	.79	.10	9.2	2.6	47	24.0	18.6	4.3
	COMBINATION	.72	.14	6.3	2.3	32	16.4	15.8	4.7
	HSP-NDT	.74	.18	4.2*	1.8	13	7.9	9.3	3.0
	HSP	.91	.12	5.7	2.3	7*	5.2	8.6*	2.9
p = .1 Binomial	DDATE	.72	.11	10.3	3.2	122	56.8	33.9	10.0
	SPT	.53*	.11	6.5	2.1	77	35.1	24.4	6.8
	SLACK	.85	.12	10.4	3.2	55	33.1	19.9	5.1
	COMBINATION	.72	.12	6.8	2.4	40	25.0	16.9	5.7
	HSP-NDT	.75	.14	6.0*	2.3	22	12.5	12.7*	3.5
	HSP	.94	.08	8.4	3.6	15*	9.6	13.1	4.2

Table 1. Tardiness Statistics for Problem 1

Random Variable x >		Fraction of Jobs Tardy		Mean Tardiness		Variance of Tardiness		Maximum Tardiness	
Processing Time Distribution	Priority Rule	$\bar{x}$	s_x	$\bar{x}$	s_x	$\bar{x}$	s_x	$\bar{x}$	s_x
Deterministic	DDATE	.67	---	5.5	---	49	---	19	---
	SPT	.67	---	9.3	---	81	---	24	---
	SLACK	.67	---	3.7	---	8	---	6	---
	COMBINATION	.67	---	3.7	---	8	---	6	---
	HSP-NDT	.67	---	3.7	---	8	---	6	---
	HSP	0*	---	0*	---	0*	---	0*	---
Uniform	DDATE	.76	.13	7.6	2.2	65	22.4	21.7	4.5
	SPT	.65	.08	7.7	1.7	71	33.5	19.4	5.6
	SLACK	.69	.05	5.4	1.0	21	7.3	10.6	2.2
	COMBINATION	.69	.06	5.1	1.3	19	8.7	9.9	2.7
	HSP-NDT	.68	.06	4.8	1.3	18	8.1	10.0	2.8
	HSP	.54*	.30	1.2*	0.9	1*	1.7	2.6*	1.7
p = .5 Binomial	DDATE	.74	.15	7.9	2.0	76	35.2	22.5	5.5
	SPT	.63*	.09	7.4	2.2	70	43.3	19.5	6.7
	SLACK	.71	.11	5.9	2.0	28	15.6	12.2	4.3
	COMBINATION	.73	.09	5.9	1.9	25	11.7	11.7	3.5
	HSP-NDT	.69	.11	5.8	2.3	25	13.4	11.8	4.2
	HSP	.67	.32	2.5*	2.1	4*	4.6	4.8*	3.2
p = .1 Binomial	DDATE	.66	.19	7.0	3.1	66	37.7	20.5	6.9
	SPT	.61*	.11	8.0	2.3	82	47.4	20.6	5.9
	SLACK	.72	.12	6.5	2.6	33	18.9	13.4	4.7
	COMBINATION	.74	.14	7.0	2.7	33	17.3	14.0	4.3
	HSP-NDT	.70	.13	6.3	2.6	30	15.8	13.1	3.9
	HSP	.78	.29	4.6*	3.1	8*	8.2	7.8*	4.3

Table 2. Tardiness Statistics for Problem 2

Random Variable x >		Fraction of Jobs Tardy		Mean Tardiness		Variance of Tardiness		Maximum Tardiness	
Processing Time Distribution	Priority Rule	$\bar{x}$	s_x	$\bar{x}$	s_x	$\bar{x}$	s_x	$\bar{x}$	s_x
Deterministic	DDATE	.36	---	1.9	---	12	---	11	---
	SPT	.43	---	3.2	---	23	---	14	---
	SLACK	.21	---	1.1	---	5	---	7	---
	COMBINATION	.29	---	1.6	---	10	---	11	---
	HSP-NDT	.36	---	1.4	---	4	---	6	---
	HSP	.07*	---	0.1*	---	0.1*	---	1*	---
Uniform	DDATE	.31	.12	2.0	1.0	15	8.6	11.9	4.0
	SPT	.48	.13	4.8	1.4	51	18.9	21.4	4.8
	SLACK	.25*	.10	1.1*	0.7	6	4.8	7.4	3.1
	COMBINATION	.49	.17	3.5	1.8	24	16.5	13.5	4.5
	HSP-NDT	.32	.13	1.5	0.9	8	7.2	8.2	4.3
	HSP	.32	.20	1.1*	0.8	3.7*	3.1	5.2*	2.3
p = .5 Binomial	DDATE	.37*	.15	2.7	1.3	26	19.0	14.9	5.7
	SPT	.50	.14	5.6	2.3	70	41.6	25.0	8.4
	SLACK	.39	.19	2.4*	1.5	14*	9.2	10.4*	3.4
	COMBINATION	.50	.15	3.5	1.4	26	17.0	14.8	5.1
	HSP-NDT	.39	.17	2.5	1.5	18	13.1	12.1	5.4
	HSP	.59	.21	3.7	2.2	16	12.5	10.6	4.5
p = .1 Binomial	DDATE	.37*	.18	3.1	1.9	30	23.1	16.0	7.0
	SPT	.51	.14	6.4	2.6	79	39.8	25.8	6.2
	SLACK	.39	.17	3.0*	2.4	25	25.3	13.2*	6.7
	COMBINATION	.53	.16	4.4	2.4	33	20.6	16.7	5.5
	HSP- NDT	.44	.19	3.1	2.0	22*	17.6	13.6	5.9
	HSP	.68	.21	5.6	3.5	31	25.8	15.8	7.2

Table 3. Tardiness Statistics for Problem 3